

## Models and methods for precise determination of ionospheric delay using GPS\*

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**Abstract** The basic principles and methods for precisely determining ionospheric delays in GPS observations are introduced and discussed. Various methods and models for fitting ionospheric delays based on GPS are compared and analyzed, and applications of the methods and models to the research and engineering are summarized.

**Keywords:** GPS, ionospheric delay, atmosphere modeling, precise GPS positioning.

It is well known that the Earth's ionosphere is a dominant error source in GPS data analyses, especially for ground-based GPS users during high solar activities or geomagnetic anomalies. Positioning precision of single-frequency GPS receivers is significantly decreased due to ionospheric delays. Although dual-frequency GPS users can conduct a self-correction for ionospheric delays using the ionosphere-free observations, it not only increases observation noise level but also makes it more difficult to separate satellite's phase center biases, and satellite's clock errors with satellite's instrumental biases in GPS data. In essence, this indirectly limits both positioning precisions and selection of positioning methods for dual-frequency GPS users.

In this regard, precisely determining ionospheric delay is required by both dual-frequency GPS users who want to further advance performance and single-frequency GPS users who need to achieve high-precision ionospheric delay correction. For these aspects it follows that although many significant research achievements have been made<sup>[1-22]</sup>, some critical issues need to be further investigated and solved. Usually, precisely establishing local ionospheric model may be achieved by using high-precision GPS data from a single reference station or a local reference network. The quality of GPS-based ionospheric delay estimates depends primarily on: (1) mathematical model and reference frame used for describing local iono-

spheric variations properties; (2) ionospheric mapping function and observation elevation; (3) characteristics of spatial distribution of ionospheric delay observation, systematic errors correction and observation precision. The crucial subjects yet to be urgently solved include: properly establish or select local ionospheric delay fitting model, accurately define and determine satellite's instrumental biases, precisely process ionospheric delay observation data, and efficiently compute ionospheric model coefficients.

In view of this, we will systematically introduce the models and methods for determining ionospheric delay using GPS data.

### 1 GPS ionosphere delay and its mathematical expression

Provided that stochastic effects of ionosphere delays are neglected and user's requirements for precisions are met, the mathematical expression of the ionospheric delay in GPS observation to the total electron contents (TEC) along the sight-of-line between receiver and satellite can be written as<sup>[13]</sup>:

$$I_i = C_i \cdot \text{TEC} = mf \cdot C_i \cdot \text{VTEC} = mf \cdot I_{i,v},$$
 where  $I_i (i = 1, 2)$  is the slant ionospheric delay;  $I_{i,v}$  the vertical ionospheric delay;  $mf$  the ionospheric mapping function; VTEC the vertical total electron content;  $C_i = 40.3/f_i^2$ ; and  $f_i$  is the  $L_i$  carrier frequency.

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It is usually assumed that all free electrons in the Earth's ionosphere are concentrated on a quasi-slim ionospheric shell at the altitude of  $H$  over the Earth's surface, and the VTEC can be parameterized using the slim ionospheric model. In theory, the ideal value of  $H$  should be as close as possible to the layer height with the maximum electron density, and 350 km, 400 km and 450 km are usually taken. Although a multi-layer ionospheric model is usually used in investigating the construction and dynamic characterization of the Earth's ionosphere, the slim model (especially a local model) has the advantage in determination of horizontal distribution of the ionospheric TEC and the provision of high-precision ionospheric delay corrections required by various related GPS engineering due to its high accuracy and operation convenience.

The effects of instrumental biases and integer ambiguities must be removed in precisely determining ionospheric delay based on zero-difference dual frequency GPS observations. Separating ionospheric delay from both instrumental biases and ambiguity parameters can be usually achieved with an ideal mapping function  $mf$ . Ionosphere modeling is usually performed using a proper ionospheric shell model for describing VTEC and a suitable slim mapping function. Various mapping functions have been applied to the GPS ionosphere research and applications. There are almost no differences among them if elevation cut-off angle of  $15^{\circ}$ – $20^{\circ}$  is adopted.

Currently two types of ionosphere models have been developed. One type is the empirical ionosphere models, such as the IRI model, the BENT model and the Klobuchar model. These models are complicated and cannot achieve high-precision ionospheric delay correction. They are not therefore suitable for high-precision GPS users. The other type is the fitting ionosphere models established based on high-precision dual-frequency GPS data. The most commonly used local ionosphere fitting model is the polynomial model (POLY). This model can usually achieve a relatively high correction precision for only a short period of time (e.g. several hours). However, the generalized trigonometric series function model (GTSF) can fit well the ionospheric delay for a long session of GPS data and can properly select ionospheric model and adjust the number of model parameters to improve ionospheric delay fitting precision. A low-order spherical harmonic model (SH) is also a comparatively good local ionospheric model, while a high-order SH model may have an obviously better accuracy than the em-

pirical models in global or regional ionosphere modeling. In this paper, several frequently-used ionospheric delay fitting models will be further analyzed and investigated.

GPS-based ionospheric TEC can be parameterized in the following reference systems: the Earth-fixed (or Solar-fixed) geographical systems and the Earth-fixed (or Solar-fixed) geomagnetic systems, i.e. in a specific reference system above, a VTEC model can be established with respect to both longitudes and latitudes which are used as the most important variables to construct the model. GPS data from a single station can be applied to monitor the ionosphere TEC over a limited range along the longitude sector. This is suitable to account for small-scale temporal/spatial TEC variations and snap ionospheric TEC at one epoch. In a Solar-fixed reference frame, the SH-based TEC is in a relatively clam/static state, and its small-scale variations are neglected accordingly. This results in a much smoothened TEC estimate, which will not significantly affect the effectiveness of GPS-based TEC determination.

## 2 The principles/methods for determination of ionospheric delay using GPS data

### 2.1 Basic principles and key issues in GPS-based TEC estimation

Usually, it is very difficult by ground GPS data alone to directly determine the ionospheric electron density variations vertically with its altitude over the Earth's surface. It may however play an important role in estimation of the total ionospheric delay caused by the path TEC along the line-of-sight between the receiver and satellite. Differential GPS data are not available for determination of local ionospheric TEC over a single station alone. But it usually has better precision than zero-difference GPS data. In addition, differential GPS data over a long baseline are only affected by relatively ionospheric delays. It is very difficult to separate the residual ionospheric delay errors from phase ambiguity parameters, which is required for precise determination of absolute ionospheric delays, and the utilization efficiency of the corresponding raw GPS data may be decreased to meet the requirements for forming the differential data. Relatively, zero-differential geometry-free GPS combination observations are more suitable for determining ionospheric TEC model, which include: the single-frequency geometry-free code and phase combination

observation ( $LP_4$ ); the dual-frequency geometry-free code combination observation ( $P_4$ ); the dual-frequency geometry-free phase combination observation ( $L_4$ ), and the dual-frequency geometry-free phase-smoothed code combination observation ( $\tilde{P}_4$ ). Both instrumental biases and integer ambiguities make it very difficult to directly determine ionospheric delay using GPS phase data alone. The accuracy and reliability of determination of ionospheric delays using dual-frequency GPS code observation depend mainly on the correction effectiveness of instrumental biases. A frequently used approach is to use a period of GPS data to determine ionosphere model parameters with the least square technique and then derive the ionospheric delay. The estimation effectiveness using the method depends mainly on data quality, properties of model construction, systematic errors processing such as instrumental biases correction, mapping function, and temporal/spatial coverage range of the ionospheric model. Hence, the key issues in precise estimation of

ionospheric delay using GPS data include: select proper ionospheric model, reduce instrumental biases in code-based ionospheric delay estimation, and ensure the precision and reliability of ionospheric delay estimation.

## 2.2 Various schemes and methods for GPS-based ionospheric delay determination

In related scientific research and engineering application, according to operational performance requirements and practical conditions, various proper schemes/methods should be designed for different ground/space-based users. Among them, the following scenarios are involved: static-time, real-time, kinematics, post-processing, and calm/abnormal states. Tens of related schemes and methods are summarized and proposed here, which are grouped into two types: basic schemes and extended schemes (Table 1).

Table 1. A comparison among different schemes for determination of ionospheric delays<sup>1)</sup>

Scheme	Observation types			Unknown parameter types			Instrumental biases processing	
	$LP_4$	$P_4(\tilde{P}_4)$	$L_4$	Ionospheric delay parameters	Ionospheric biases	Ambiguity parameters	Considering this error	Different methods for estimating this error
Basic schemes								
1			y	y	y	y	y	Along with ambiguity
2	y			y	y	y	y	Along with ambiguity
3		y		y	y		y	Direct computation
4		y		y				
5		(y)		y				
6		(y)		y	y		y	Direct computation
Extended schemes								
7		y		y			y	Using Scheme 1
8		y		y	y		y	Using Scheme 3
9		(y)		y			y	Using scheme 3
10		y		y			y	Using Scheme 4
11		(y)		y			y	Using Scheme 4
12			y	y	y	y	y	Using Scheme 5
13			y	y			y	Using Schemes 6 and 4

1) y = yes

### 2.2.1 Basic fitting schemes

**Scheme 1.** Using dual-frequency phase combination  $L_4$ .

Tracking ionospheric delay from the dual-frequency geometry-free phase combination observation  $L_4$  is a high-precision post-processing method, whereby ionospheric delays are determined along with the

corresponding combination terms consisting of both instrumental biases and integer ambiguities named clk. This scheme is usually applied to determine local/regional/ global ionospheric delay model parameters, with all cycle slips being efficiently repaired or processed. It should be noted that this scheme usually makes it difficult to process a large number of clk parameters when monitoring ionospheric TEC over a large coverage area using a long period of GPS data.

**Scheme 2.** Using single-frequency code and phase combination  $LP_4$ .

Tracking ionospheric delay from the single-frequency geometry-free code and phase combination observation  $LP_4$  is performed along with estimation of combination terms consisting of both instrumental biases and ambiguities. This scheme is usually considered as a candidate method for determining ionospheric delays from a dual-frequency GPS reference station. For this scheme it follows that all cycle slips must be efficiently repaired as well, which is more difficult than Scheme 1. A larger number of instrumental biases parameters needs to be coped with when implementing this scheme.

**Scheme 3.** Using dual-frequency code combination  $P_4$ .

Tracking ionospheric delay from dual-frequency geometry-free code combination observation  $P_4$  is performed along with estimating instrumental biases. This scheme is also frequently used for determining ionospheric delay model parameters with dual-frequency GPS data. The number of instrumental biases is the same as that of the observed satellites. However, under the same conditions, the number of instrumental biases is much less than that of the clk parameters in Scheme 1. Compared to Scheme 1, Scheme 3 has a better reliability, less computational task and a little bit lower precision.

**Scheme 4 (Scheme 5).** Using dual-frequency code combination  $P_4$  (dual-frequency phase-smoothed code combination observation  $\tilde{P}_4$ ) to determine ionospheric model parameters without considering instrumental biases.

Tracking ionospheric delay from dual-frequency geometry-free code combination observation  $P_4$  (dual-frequency phase-smoothed code combination observation  $\tilde{P}_4$ ) is performed without estimating instrumental biases terms, i. e. the instrumental biases are considered as a part of ionospheric delays. This limits to some extent the ionospheric delay estimation precision. But this scheme, along with Scheme 3 or 6, can be applied to analyzing and assessing in quantity characteristics and levels of the effects of instrumental biases.

**Scheme 6.** Dual-frequency phase-smoothed code combination observation  $\tilde{P}_4$ .

Tracking ionospheric delay from dual-frequency phase-smoothed code combination observation  $\tilde{P}_4$  is performed along with estimating instrumental biases. Precision using Scheme 6 is higher than that using Scheme 3, and its reliability is better than Scheme 1. Scheme 6 has also a high computation efficiency and superior performance. Thus it is considered as one of the best methods for ionospheric delays estimation.

### 2.2.2 Extended fitting methods

Using the above methods for determining ionospheric delays and with the related characteristics of GPS ionosphere observations, various schemes can be applied to efficiently determine instrumental biases and ambiguity parameters along with their combination terms. To meet the requirements for related researches and applications, several methods for determining instrumental biases and integer ambiguities have been developed by us, including:

(1) To estimate ionospheric model parameters using Scheme 1, determine ionospheric delay according to  $I_1 = mf \cdot I_{1,v}$ , and then insert the estimated  $I_1$  into dual-frequency code observations to obtain the individual daily-average values of instrumental biases for all the observed satellites.

(2) Substituting  $I_1$  into dual-frequency phase-smoothed code observation to obtain the individual daily-average values of instrumental biases for all observed satellites.

(3) Substituting  $I_1$  into dual-frequency code observation to obtain the individual session-average values of instrumental biases for all the observed satellites according to the different numbers of phase ambiguities over different observation sessions.

(4) Substituting  $I_1$  into dual-frequency phase-smoothed code observation to obtain the individual session-average values of instrumental biases for all the observed satellites according to the different numbers of phase ambiguities over different observation sessions.

(5) Differencing between dual-frequency phase observations and dual-frequency code observations to obtain the individual session-average values of instrumental biases for all the observed satellites according to the different numbers of phase ambiguities over different observation sessions.

(6) Differencing between dual-frequency phase observations and dual-frequency phase-smoothed code observations to get the individual session-average values of instrumental biases for all the observed satellites according to the different numbers of phase ambiguities over different observation sessions.

Based on these methods for fitting ionospheric delays along with processing instrumental biases and ambiguities, the following new schemes are designed for further improving ionospheric delays estimation.

**Scheme 7 (Scheme 8).** Using dual-frequency code combination  $P_4$  to determine ionospheric model parameters and reducing instrumental biases by the approaches described in (1) and (3).

Firstly, correcting the instrumental biases in dual-frequency code combination  $P_4$  with their estimates obtained in (1) and (3), then using the reduced-combination observations  $P_4$  (they both are considered as two kinds of estimates of vertical ionospheric delay  $I_{4,v}$ ) to determine ionospheric delay model parameters. From the daily/session-based predictability of the estimated instrumental biases, the real-time estimation of ionospheric model parameters or ionospheric delays can be achieved in a simple way.

**Scheme 9 (Scheme 10).** Using dual-frequency phase-smoothed code combination  $\tilde{P}_4$  (dual-frequency code combination  $P_4$ ) to determine ionospheric model parameters, and reducing instrumental biases by the approaches described in (3) and (4).

Firstly, correcting the instrumental biases in dual-frequency code combination  $\tilde{P}_4(P_4)$  with their estimates obtained in (3) and (4), then using the reduced-combination observations  $\tilde{P}_4(P_4)$  (they both are two kinds of vertical ionospheric delay estimates  $I_{4,v}$ ) to determine ionospheric delay model parameters. From the daily/session-based predictability of the estimated instrumental biases, these two schemes can also achieve the real-time estimation for ionospheric model parameters or ionospheric delays in a simple way, with an operational performance similar to Schemes 7 and 8 and better precision and reliability.

**Scheme 11.** Using dual-frequency phase-smoothed code combination  $\tilde{P}_4$  to determine ionospheric model parameters, and reducing instrumental biases in the way described in (4).

Firstly, correcting the instrumental biases in dual-frequency code combination  $\tilde{P}_4$  with their estimates obtained in (4), then using the reduced  $\tilde{P}_4$  combination observations (it is also an estimate of vertical ionospheric delay  $I_{4,v}$ ) to determine ionospheric delay model parameters. This scheme can also achieve a session-based prediction for instrumental biases, with the same operational performance in practice as Scheme 10 and better determination effectiveness due to the higher precision and reliability of the observations  $\tilde{P}_4$  with respect to  $P_4$ .

**Scheme 12.** Using dual-frequency phase combination  $L_4$  to determine ionospheric model parameters and instrumental biases, and reducing the combination term consisting of both instrumental bias and ambiguity in the way described in (5).

Firstly, correcting both the instrumental biases and ambiguities in dual-frequency phase combination  $L_4$  with their estimates obtained in (5), then using the reduced-combination observations  $L_4$  to determine ionospheric delay model parameters and instrumental biases. This scheme can be implemented once an hour, usually with a relatively high precision and reliability in determination of ionospheric delay. It is primarily applied to testing data-processing quality and evaluating estimation effectiveness of instrumental biases and ambiguities obtained from other approaches.

**Scheme 13.** Using dual-frequency phase combination  $L_4$  to determine ionospheric model parameters, and reducing combination term consisting of instrumental bias and ambiguity in the ways described in (4) and (6).

Firstly, correcting the instrumental biases and integer ambiguities in dual-frequency phase combination  $L_4$  with their estimates obtained in (4) and (6), then using the reduced-combination observations  $L_4$  to determine ionospheric delay model parameters. Similar to Scheme 12, this scheme can be implemented once an hour, usually with a relatively high precision and reliability in determining ionospheric delay. It is also primarily applied to testing data-processing quality and verifying effectiveness of estimating instrumental biases and integer ambiguities using other approaches.

### 3 Experimental results and analyses

#### 3.1 Experimental contents and methods

With GPS observations in which gross errors and

cycle slips have been mitigated and repaired, we have tested all the aforementioned schemes, methods and models for their operational performance, fitting effectiveness and applications in establishing mathematical expression and precisely modeling ionosphere TEC. The experimental contents and methods include:

(1) Scheme for analyzing the effects of reference systems on modeling ionospheric delay.

With different reference systems, such as the Earth-fixed geographical/geomagnetic system and the Solar-fixed geographical/geomagnetic system, we determined an identical set of parameters for a specific ionospheric model (such as the PLOY model, the GTSF model or the SH model), then analyzed and investigated the effects of the different reference systems on ionospheric delay determination using different ionospheric models to ensure that a relatively suitable reference system can be properly selected.

(2) Scheme for analyzing the effects of longitudes/latitudes on modeling ionospheric delay.

Selecting a set of reference stations with close longitudes (or latitudes) and different latitudes (or longitudes) to determine individual parameters for a specific ionosphere model using GPS data from each station, then illustrating in universal time (UT) the differences among the ionospheric TEC estimates corresponding to different stations, and analyzing the characteristics of variations with geographic longitudes (or latitudes) of the different ionospheric TEC models to know their modeling performance in quantity.

(3) Scheme for investigating dominant variation characteristics of ionospheric TEC.

Selecting some reference stations operating in different areas to determine an identical ionospheric model using different sets of multi-day continuous real GPS data which correspond to the different stations, then analyzing and investigating the multi-day continuous variation characteristics of the ionospheric TEC estimates, and comparing the differences among the variation properties of the ionospheric TEC estimates obtained by using different models in different areas.

(4) Scheme for comparing performances of modeling ionosphere TEC of different models.

With the selected reference systems, combining

identical GPS data and fitting methods with different ionospheric models to determine ionospheric TEC, and then comparing the differences among the estimation precisions of ionospheric TEC to evaluate individual modeling effectiveness for different ionospheric models.

(5) Scheme for analyzing the fitting precisions of ionosphere delays for different models.

With the selected reference systems, combining an identical set of GPS data and a specific ionospheric model with different fitting methods to determine different estimates of ionospheric TEC, and then comparing precisions of the ionospheric TEC estimates to analyze individual effectiveness of ionospheric delay determination for different fitting methods.

(6) Scheme for analyzing the effects of systematic errors in ionospheric observations.

Analyzing the effects of instrumental biases on ionospheric delay determination and the corresponding correction methods and effectiveness in different fitting schemes. With the selected reference systems, combining an identical set of GPS data and a specific ionosphere delay model with different fitting methods to determine ionospheric TEC, and then comparing precisions of the ionospheric TEC estimates to analyze the respective effectiveness of ionospheric delay determination for different fitting methods.

(7) The above mentioned selected stations include the stations with identical latitudes (BJSH, DXIN and TASH), and the stations with identical longitudes (HLAR, TAIN and WUHN).

(8) Definition of reference systems for describing the ionosphere mentioned above.

The Solar-fixed geographical system 1(2): Both the geographical latitude of the point at which VTEC is to be established and the difference between geographical longitudes of this point and the mean (true) sun are as the dominant variables, and applied to establishing ionospheric VTEC model.

The Solar-fixed geomagnetic system 1(2): Both the geomagnetic latitude of the point at which VTEC is to be established and the difference between geomagnetic longitudes of this point and the mean (true) sun are as the dominant variables, and applied to establishing ionospheric VTEC model.

### 3.2 Experimental results and analyses

In this work, the Earth's ionosphere was considered as a quasi-slim spherical shell at the altitude of 350 km over the Earth's surface, and the Earth's radius ( $R_e$ ) was 6371.3951 km. All TEC values were transformed to the corresponding ionospheric delays in L1 carrier signals. The GPS data of 7 days in a row (some data missing at a few stations) from GPS reference stations of the Crust Movement Observation Network Of China (CMONOC) were analyzed. Observation time span was from August 30, 1998 to September 5, 1998. Data sampling interval was 30s, and the elevation cut off angle was 20 degrees. Experimental results are illustrated in Table 2 and Figs. 1—4. Figs. 1 and 2 show the results obtained using the PLOY model, the GTSF model and the SH model with geographical reference systems. Figs. 3 and 4 demonstrate the results obtained using the SH model with the geomagnetic reference system alone and with all the above four reference systems, respectively. Table 2 lists the internal precisions of the fitted ionospheric TEC by combining the 13 schemes with the SH model in the Solar-fixed geographic system.

In Figs. 1—3, the  $x$ -axis is the local time at the corresponding station, in which 24—35 h represents 0—11 h on the next day, and the  $y$ -axis is vertical ionospheric delays in units of meters. Figs. 1—3 demonstrate the difference in the ionospheric TEC estimates obtained by different ionospheric models in a reference system, and the difference in the ionospheric TEC estimates obtained by a specific model (e.g. the SH model) in different reference systems (e.g. the four systems mentioned previously). From Figs. 3 and 4, it can be seen that the difference in modeling characteristics comes from the coordinate reference system used, namely the geographic or geomagnetic reference system, rather than the time reference used, namely the mean or true solar time reference, therefore different reference systems have different capabilities of representing variations of the Earth's ionosphere TEC, and selection of a reference system should consider both the solar and geomagnetic effects. Usually, one may select a solar-fixed geomagnetic system with a mean solar time reference for modeling ionosphere TEC, while the resulting TEC estimates can be shown in a solar-fixed geographic reference system. Hence, for GPS-based ionospheric TEC determination, the two following factors should be considered as much as possible: computation effi-

ciency and reference system performance.

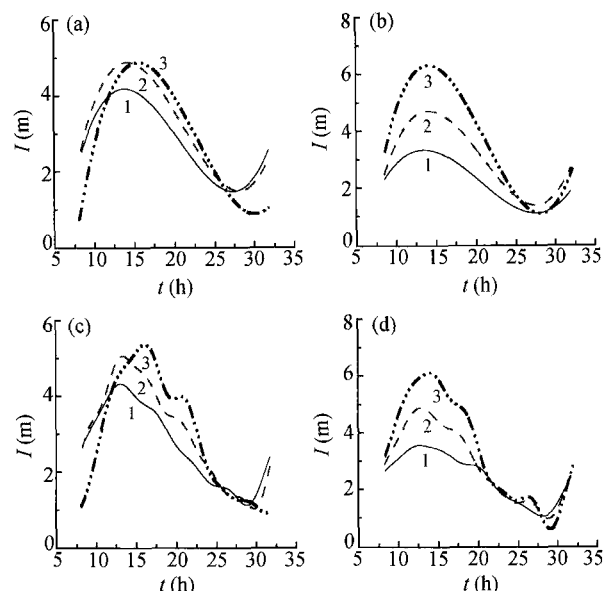


Fig. 1. Time vs vertical ionospheric delays. (a) Using the POLY model and the data from stations at similar latitudes; (b) using the POLY model and the data from stations at similar longitudes; (c) using the GTSF model and the data from stations at similar latitudes; (d) using the GTSF model and the data from stations at similar longitudes.

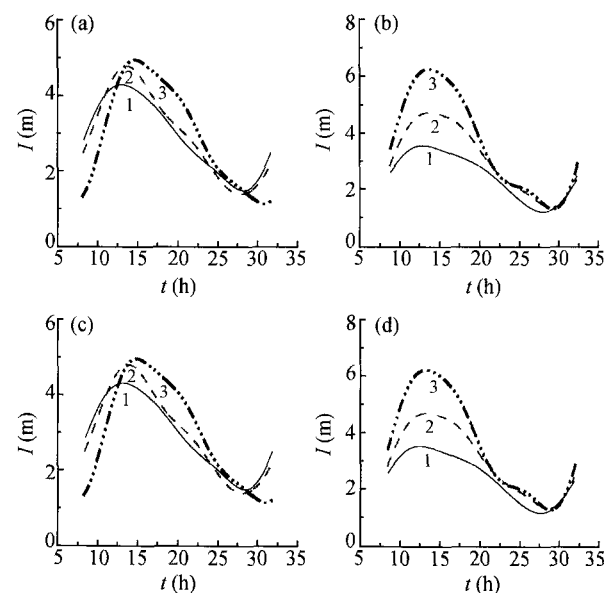


Fig. 2. Variations of time vs vertical ionospheric delays obtained using the SH model in the Solar-fixed geographic reference systems. (a) Solar-fixed geographic reference system 1, the stations at similar latitudes; (b) solar-fixed geographic reference system 1, the stations at similar longitudes; (c) solar-fixed geographic reference system 2, the stations at similar latitudes; (d) solar-fixed geographic reference system 2, the stations at similar longitudes.

From Figs. 1—3, it can be found that no matter what reference system is used, a specific ionosphere model and the individual GPS data from the three stations at similar longitudes but different latitudes produce a similar variation of TEC values, that means,

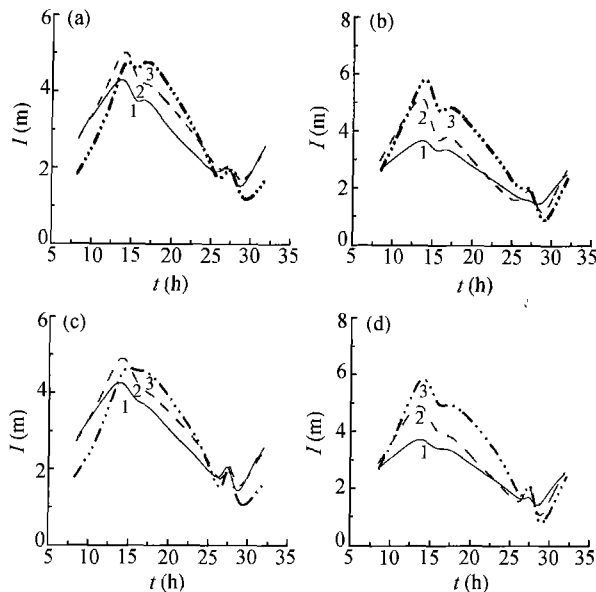


Fig. 3. Variations of time vs vertical ionospheric delays obtained using the SH model in Solar-fixed geomagnetic reference systems. (a) Solar-fixed geomagnetic reference system 1, the stations at similar latitudes; (b) solar-fixed geomagnetic reference system 1, the stations at similar longitudes; (c) solar-fixed geomagnetic reference system 2, the stations at similar latitudes; (d) solar-fixed geomagnetic reference system 2, the stations at similar longitudes.

Table 2. Root mean square (RMS) of vertical ionospheric delay residual errors (in L1 carrier) estimated using different fitting methods and the SH model in the solar-fixed reference system 1 (unit: m)

Schemes	Days						
	242	243	244	245	246	247	248
1	0.16	0.24	0.27	0.25	0.25	0.17	0.25
2	0.19	0.25	0.24	0.26	0.26	0.17	0.24
3	0.38	0.44	0.41	0.44	0.43	0.19	0.29
4	1.14	1.15	1.14	1.17	1.17	1.12	0.89
5	1.10	1.10	1.10	1.12	1.13	1.12	0.91
6	0.17	0.28	0.25	0.28	0.26	0.17	0.28
7	0.38	0.44	0.42	0.44	0.43	0.22	0.31
8	0.17	0.25	0.22	0.26	0.25	0.16	0.30
9	0.37	0.42	0.40	0.42	0.42	0.19	0.27
10	0.38	0.43	0.40	0.43	0.42	0.19	0.27
11	0.16	0.25	0.22	0.26	0.25	0.16	0.29
12	0.17	0.27	0.24	0.27	0.26	0.18	0.28
13	0.16	0.24	0.22	0.25	0.25	0.17	0.25

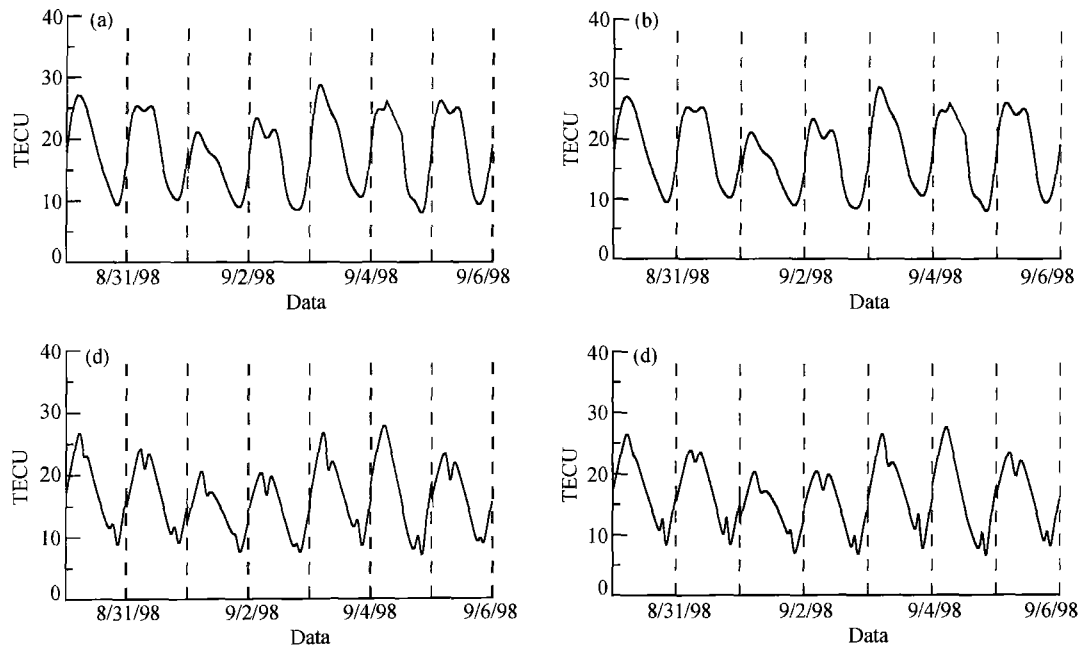


Fig. 4. Variations of the BJFS's vertical ionospheric TEC estimated using the SH model. Time period: GPS week number 973, 1998 (i.e. 8/30/1998–9/5/1998). (a) and (b) denote respectively the Solar-fixed geographic reference system 1 and 2; (c) and (d) denote respectively the Solar-fixed geomagnetic reference system 1 and 2.

the occurrence time of the maximum/minimum TEC values is almost the same while the corresponding TEC values are different to some extent. These imply that for fitting ionospheric TEC by using of a specific model and a set of GPS data, both the size and the

synchronic property of the TEC estimates are closely correlated with latitudes and longitudes. This is referred to as longitude effect in the ionospheric TEC estimation using GPS data and the mathematical models, and all the models used here have demon-



strated their dominant characteristics. Also the ionosphere model construction limits to some extent the ionospheric TEC estimation since it fits background TEC only in a deterministic sense.

From Fig. 4, it can be found that the SH-fitted ionospheric TEC estimates vary diurnally in all reference systems (the other two types of results are not given here due to limited space requirement for this paper). But the difference in the variation property of the ionospheric TEC estimated between the solar-fixed geographic and geomagnetic systems can also be observed. Especially there is a double-peak phenomenon for the ionospheric TEC estimates in the geomagnetic reference system. Further analysis and verification of whether this is the “true” ionospheric state at that time and whether it is caused by the reference system used are needed, based on a larger amount of GPS data. In addition, in the Solar-fixed geographic reference system, the TEC estimates vary relatively smoothly with time, while in the Solar-fixed geomagnetic reference system, the TEC estimates show more detailed characteristics of its relatively irregular variations, and they also have better continuity than those in Solar-fixed geographic reference system, which is more advantageous to describing ionospheric TEC variations properties. Comparing the results given in Ref. [15], it can be seen that all the ionospheric TEC estimates obtained by the three models vary continuously as a whole, while the GTSF has a relatively better continuity than the SH model, and the latter is superior to the POLY model.

From Figs. 1 and 2, one can see that in a specific reference system (e.g. a solar-fixed geographic reference system), the variation properties of the three types of TEC estimates obtained using the above three models are different. Table 2 lists the internal precisions of the three TEC estimates. Although the limited data used here make it impossible to draw a quantitative conclusion for the modeling performance of the three ionospheric models, one can see that they have different modeling performances, i.e. different mathematical models have different capabilities of modeling ionosphere TEC variation properties. For the three ionospheric models used in this paper, the POLY model relatively suits well for fitting ionospheric delays only in a short time period of several hours<sup>[15]</sup>. The GTSF model should be applied to daily-based ionospheric delay determination or fitting the ionospheric delays over a long time span (multi-day) of

GPS data, especially in mid-latitude areas<sup>[15]</sup>, and the SH model can also be used in this case as a second selection. The SH model is one of the best candidates for representing regional or global ionospheric TEC<sup>[16,17]</sup>. For some large range GPS application systems, such as the FAA’s WAAS, the SH model is also an ideal selection for ionospheric TEC modeling. It has been however investigated that combining the GTSF model with the idea “different areas for different stations” (DADS), an ionospheric grid delay model with a better precision and accuracy can be established<sup>[18–20]</sup>.

Table 2 lists the internal precisions of the ionospheric TEC estimates using the SH model and the 13 types of fitting methods. One can see that all the results without reducing instrumental biases are obviously worse. In fact, these methods are not applied in practice, and they are usually used to compare and analyze the effects of instrumental biases on ionospheric delay determination. The other results, reduced with proper instrumental biases estimates, have much higher internal precisions if data pre-processing has been performed successfully. But for these methods, there are different performances, accuracies, and characteristics in practical applications. They are therefore suitable for different applications and researches.

Compared all the results in Table 2, it can also be found that the internal precision of the ionospheric delay estimates using Scheme 1 is closest to that using Scheme 13, and each of the other extended methods can achieve higher fitting precisions, which is closer to Scheme 1 than all the aforementioned basic methods. These indicate that all the extended methods/schemes involved in this paper are successful in coping with instrumental biases and improving observation precisions. The phase-based smoothing techniques, proper selection of reference system and decrease of observation noise level are advantageous to improving dual-frequency code-based ionospheric TEC determination precision. Among them, the instrumental bias is the key factor which causes the severest effects on ionospheric delay determination and must be reduced as much as possible. In fact, the ionospheric delay estimates may achieve similar precisions for all the methods mentioned above if the instrumental biases are efficiently corrected for.

In addition, Table 2 also demonstrates that the precisions of the TEC estimates using different meth-

ods and a specific ionospheric model are different from one to the other. In fact, there exist the similar differences among the precisions of the TEC estimates using the three ionospheric models (the results using the other two models are not given here). If data pre-processing is successful, from the point of view of requirements for computation method and software making, it is intended that the basic methods are easily implemented in a simple way, the extended methods can improve the estimation precision of the TEC but their implementations are relatively complicated, and the precisions of the ionospheric delay estimates using a specific fitting method and different ionospheric models are different as well.

## 4 Conclusions

The following conclusions can be drawn from the above investigation: (1) Based on quasi-shell assumption and slim-ionosphere model, precise determination of ionospheric delays by selecting proper ionosphere delay model and mapping function, parameterizing vertical ionospheric TEC delays, reducing instrumental biases which are regarded as system errors, and using the least squared technique to extract ionospheric delays from a period of GPS data, especially dual-frequency phase (or phased-smoothed code) data, is a feasible approach. (2) Different mathematical models and reference systems have different capabilities of representing ionospheric TEC over a specific area, and for different mathematical models and reference systems, the effectiveness of representing ionospheric TEC over different areas is different. (3) For a specific model and reference system, the precision of ionosphere TEC estimation using different fitting methods is also different, which means that selecting a method available for ionosphere delay determination should consider as much as possible user's requirements for both accuracies and operational performances. (4) Instrumental bias must be effectively reduced since it is the dominant error source in GPS-based ionosphere delays determination.

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